Esercizio 14

Risolvere la seguente equazione:

$$\frac{2x-1}{1-x}:\frac{6+2x}{x^2-1}+\frac{x^2+2x-3}{x^2+4+4x}:\frac{x^2+9+6x}{x^2+3x+2}=\frac{5-2x^2}{4+2x}$$

Svolgimento

Raccogliamo e scomponiamo:

$$\frac{2x-1}{1-x}\cdot\frac{(x+1)(x-1)}{2(3+x)}+\frac{x^2+2x-3}{(x+2)^2}\cdot\frac{x^2+3x+2}{(x+3)^2}=\frac{5-2x^2}{2(2+x)}$$

Campo di esistenza:

$$x\neq 1; x\neq -2; x\neq -3$$

Scomponiamo, se possibile, i trinomi di secondo grado:

$$x^2+2x-3=0$$

$$x_{1-2}=-1\pm\sqrt{1+3}=-1\pm\sqrt{4}=-1\pm 2$$

$$x_1=1;\quad x_2=-3$$

Quindi:

$$x^2+2x-3=(x-1)(x+3)$$

Procediamo con il secondo trinomio:

$$x^2+3x+2=0$$

$$x_{1-2}=\frac{-3\pm\sqrt{9-4\cdot 2}}{2}=\frac{-3\pm\sqrt{9-8}}{2}=\frac{-3\pm 1}{2}$$

$$x_1=-1;\quad x_2=-2$$

Quindi:

$$x^2+3x+2=(x+1)(x+2)$$

Sostituiamo:

$$\frac{1-2x}{x-1}\cdot\frac{(x+1)(x-1)}{2(3+x)}+\frac{(x-1)(x+3)}{(x+2)^2}\cdot\frac{(x+1)(x+2)}{(x+3)^2}=\frac{5-2x^2}{2(2+x)}$$

$$(1-2x)\cdot\frac{x+1}{2(3+x)}+\frac{x-1}{x+2}\cdot\frac{x+1}{x+3}=\frac{5-2x^2}{2(2+x)}$$

Denominatore comune:

$$\frac{(1-2x)(x+1)(x+2)+2x^2-2}{2(x+3)(x+2)}=\frac{(5-2x^2)(x+3)}{2(2+x)(x+3)}$$

$$\frac{(x+1-2x^2-2x)(x+2)+2x^2-2}{2(x+3)(x+2)}=\frac{5x+15-2x^3-6x^2}{2(2+x)(x+3)}$$

$$(1-2x^2-x)(x+2)+2x^2-2-5x-15+2x^3+6x^2=0$$

$$x+2-2x^3-4x^2-x^2-2x+2x^2-17-5x+2x^3+6x^2=0$$

$$3x^2-6x-15=0$$

$$x_{1-2}=\frac{3\pm\sqrt{9+3\cdot 15}}{3}=\frac{3\pm\sqrt{9+45}}{3}=\frac{3\pm\sqrt{54}}{3}=\frac{3\pm\sqrt{9\cdot 6}}{3}=\frac{3\pm 3\sqrt{6}}{3}=$$

$$=\frac{3(1\pm\sqrt{6})}{3}=1\pm\sqrt{6}$$

$$x_1=1+\sqrt{6};\quad x_2=1-\sqrt{6}$$

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