

Esercizio 2

Calcolare:

$$\lim_{n \rightarrow +\infty} \left\{ [n^2 + \ln(n!) + \cos n] \left[\sin\left(\frac{1}{n}\right) \ln(n+1) - \operatorname{arctg}\left(\frac{1}{n}\right) \ln(n-1) \right] \right\}$$

Svolgimento

Raccogliamo n^2 :

$$\lim_{n \rightarrow +\infty} \left\{ n^2 \left[1 + \frac{\ln(n!)}{n^2} + \frac{\cos n}{n^2} \right] \left[\sin\left(\frac{1}{n}\right) \ln(n+1) - \operatorname{arctg}\left(\frac{1}{n}\right) \ln(n-1) \right] \right\} =$$

Ma:

$$\lim_{n \rightarrow +\infty} \frac{\ln(n!)}{n^2} = 0 \quad e \quad \lim_{n \rightarrow +\infty} \frac{\cos n}{n^2} = 0$$

Sostituendo:

$$\begin{aligned} &= \lim_{n \rightarrow +\infty} \left\{ n^2 \left[\sin\left(\frac{1}{n}\right) \ln(n+1) - \operatorname{arctg}\left(\frac{1}{n}\right) \ln(n-1) \right] \right\} = \\ &= \lim_{n \rightarrow +\infty} \frac{\sin\left(\frac{1}{n}\right) \ln(n+1) - \operatorname{arctg}\left(\frac{1}{n}\right) \ln(n-1)}{\frac{1}{n^2}} = \\ &= \lim_{n \rightarrow +\infty} \left[\frac{\sin\left(\frac{1}{n}\right) \ln(n+1)}{\frac{1}{n^2}} - \frac{\operatorname{arctg}\left(\frac{1}{n}\right) \ln(n-1)}{\frac{1}{n^2}} \right] = \\ &= \lim_{n \rightarrow +\infty} \left[\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \cdot \frac{\ln(n+1)}{\frac{1}{n}} - \frac{\operatorname{arctg}\left(\frac{1}{n}\right)}{\frac{1}{n}} \cdot \frac{\ln(n-1)}{\frac{1}{n}} \right] = \end{aligned}$$

Ma:

$$\lim_{n \rightarrow +\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = 1 \quad e \quad \lim_{n \rightarrow +\infty} \frac{\operatorname{arctg}\left(\frac{1}{n}\right)}{\frac{1}{n}} = 1$$

Sostituendo:

$$\begin{aligned} &= \lim_{n \rightarrow +\infty} \left[\frac{\ln(n+1)}{\frac{1}{n}} - \frac{\ln(n-1)}{\frac{1}{n}} \right] = \lim_{n \rightarrow +\infty} [n \ln(n+1) - n \ln(n-1)] = \\ &= \lim_{n \rightarrow +\infty} [\ln(n+1)^n - \ln(n-1)^n] = \lim_{n \rightarrow +\infty} \left[\ln\left(\frac{n+1}{n-1}\right)^n \right] = \end{aligned}$$

Voglio arrivare a scrivere la funzione di cui devo calcolare il limite nella forma $\left(1 + \frac{1}{f(n)}\right)^{f(n)}$.

Sottraggo e aggiungo 1 al denominatore:

$$\begin{aligned}
 &= \lim_{n \rightarrow +\infty} \left[\ln \left(\frac{n+1-1+1}{n-1} \right)^n \right] = \lim_{n \rightarrow +\infty} \left[\ln \left(\frac{n-1}{n-1} + \frac{2}{n-1} \right)^n \right] = \\
 &= \lim_{n \rightarrow +\infty} \left[\ln \left(1 + \frac{1}{\frac{1}{2}(n-1)} \right)^n \right] =
 \end{aligned}$$

Divido e moltiplico l'esponente per 2:

$$\begin{aligned}
 &= \lim_{n \rightarrow +\infty} \left[\ln \left(1 + \frac{1}{\frac{1}{2}(n-1)} \right)^{\frac{n}{2} \cdot 2} \right] = \lim_{n \rightarrow +\infty} \left[2 \ln \left(1 + \frac{1}{\frac{1}{2}(n-1)} \right)^{\frac{n}{2}} \right] = \\
 &= \lim_{n \rightarrow +\infty} \left\{ 2 \ln \left[\left(1 + \frac{1}{\frac{1}{2}(n-1)} \right)^{\frac{n}{2}} \cdot \frac{\left(1 + \frac{1}{\frac{1}{2}(n-1)} \right)^{\frac{1}{2}}}{\left(1 + \frac{1}{\frac{1}{2}(n-1)} \right)^{\frac{1}{2}}} \right] \right\} = \\
 &= 2 \lim_{n \rightarrow +\infty} \ln \left[\left(1 + \frac{1}{\frac{1}{2}(n-1)} \right)^{\frac{n}{2} \cdot \frac{1}{2}} \cdot \left(1 + \frac{1}{\frac{1}{2}(n-1)} \right)^{\frac{1}{2}} \right] = \\
 &= 2 \lim_{n \rightarrow +\infty} \left[\ln \left(1 + \frac{1}{\frac{1}{2}(n-1)} \right)^{\frac{1}{2}(n-1)} + \frac{1}{2} \ln \left(1 + \frac{1}{\frac{1}{2}(n-1)} \right) \right] =
 \end{aligned}$$

Ricordando che:

$$\begin{aligned}
 \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{f(n)} \right)^{f(n)} &= e \text{ se } \lim_{n \rightarrow +\infty} f(n) = \pm\infty \\
 &= 2 \ln e = 2
 \end{aligned}$$

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Matilde Consales